Things of science

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GEOMETRIC MODELS

Unit No. 257

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GEOMETRIC MODELS UNIT

This unit of THINGS *of science* consists of 10 specimens needed to make four models for geometry, and this leaflet.

So many high school and college students memorize theorems of geometry without actually visualizing what they mean that this series of models was developed for THINGS *of science* members with the cooperation of the National Council of Teachers of Mathematics to illustrate a variety of ideas and theorems from plane geometry. If you have already mastered plane geometry, these will help refresh your memory; if geometry is still something you will study in the future, these will help you understand some of the fundamentals. And in any case, they are interesting models to play with.

The flexible models all deal with triangles, which are simply figures with three straight lines as sides.

You will first want to cut your elastic into the following pieces: one piece seven inches long and one piece five inches long; two pieces five-and-a-half inches, and two pieces three-and-a-half inches long.

Now identify the various specimens needed to build your four models. The cardboard rectangles, around which you

will want to arrange your pieces, have the names of the models printed on them to help you tell them apart.

TRIANGLES OF EQUAL AREA — Cardboard, seven-inch piece of elastic, push pin.

MEDIAN OF A TRIANGLE—Cardboard with holes punched in it, two lengths of elastic (one five-and-a-half inches, the other three-and-a-half inches), screw eye, one-inch wooden rod.

MIDPOINTS OF SIDES—Cardboard, two pieces of elastic (one three-and-a-half inches long, the other five inches long), wooden rod two-and-a-half inches long, upholstery nail.

CIRCLE AROUND TRIANGLE — Cardboard, piece of elastic five-and-a-half inches long.

TRIANGLES OF EQUAL AREA

Experiment 1—To make a really permanent model, it is well to back the colored cardboard with heavy corrugated board that is smooth on both sides, or with very stiff cardboard cut from a packing box. Cut the corrugated board to exactly the same size as the colored cardboard and carefully tape the two together.

With your map pin or some other

pointed instrument, punch holes through both the cardboard and the corrugated board at A and B. Thread the elastic through these holes, leaving a generous loop about four and a quarter inches long on the front side of the model. Tie double knots on the reverse side so the elastic will not pull out. Ink a dotted line in black from A to B. Your model is now ready for use.

Experiment 2—Loop your elastic around the push pin and stick the pin at some point along the top line, well to the right of C. If the elastic is a bit too long to form a triangle with straight sides, shorten it from the back. Does the three-sided figure, made by the elastic and the dotted line AB, seem to have about the same area as triangle ABC?

Experiment 3—Try sticking the map pin at other points along the upper of the two parallel lines. Notice that when the pin is placed far to the right, the triangle is quite long and thin; when placed near C it becomes short and fat. In all cases the area of the triangle is equal to the area of ABC, the triangle already printed on the colored model. This is true because triangles with equal bases and equal altitudes are equal in area.

Experiment 4—Try varying the length of the sides to make the more familiar types of triangles. These are:

Scalene triangle—no two sides equal in length.

Isosceles triangle—two sides equal.

Equilateral triangle—all three sides equal.

To make the first two triangles, the push pin may still be stuck along the upper line; for the equilateral triangle it should be located below the top parallel line and within the triangle ABC.

Experiment 5—Triangles may also be classified as to the angles they contain:

Acute triangle—all three angles acute (smaller than a right angle, which is a 90-degree angle made by two lines perpendicular to each other).

Equiangular triangle—all three angles equal.

Right triangle—one right angle.

Obtuse triangle—one obtuse angle (greater than a right angle).

If you insert your pin so the elastic goes straight up at A or B, you will have a right triangle. By moving the pin either to the right of the line perpendicular to B or to the left of the line perpendicular to A, you will form a triangle with two

acute angles and one obtuse angle. If you place the pin between these two points, your triangle will have three acute angles.

Experiment 6—An oblique triangle contains no right angle. Which of the triangles you have just made are oblique?

Experiment 7—Using your elastic and push pin, how many triangles can you make of exactly the same shape and size as triangle ABC? A triangle with corresponding sides and angles equal to ABC is said to be congruent to that triangle.

MEDIAN OF A TRIANGLE

Experiment 8—To make your model, first back the colored cardboard with corrugated board or stiff cardboard. Now insert the screw eye into the wooden rod or dowel: a hole has been bored into the dowel to give you a start. At A, B and M punch holes just large enough to thread your elastic to the back side. Tie a double knot at one end of the longer elastic and thread through at A.

Run the elastic through the screw eye, thread through B and knot on the back side. The portion of the elastic showing on the front of the model should be about three and a half inches long. Now thread the shorter piece of elastic through the

screw eye and tie securely at one end. Push the long end of the elastic through the hole at M and knot, allowing about an inch and a quarter of elastic to show. Be sure the elastic is not drawn too taut when you stick the peg into the holes.

Experiment 9—Move the peg from one to another of the holes punched into the cardboard and name the various types of triangles formed by the longer of the elastics and the line AB.

Experiment 10—Watch what the elastic from M (the midpoint between A and B) to the screw eye, which marks the vertex of the angle opposite the base, does to the triangle. Only in one position does it divide the large triangle into congruent triangles and only in one position does it bisect the vertex angle opposite the base, but it always divides the figure into two triangles of equal area.

Experiment 11—A median of a triangle is a line joining any vertex to the midpoint of the opposite side. Insert the peg into the hole at far left or far right, and with a pencil lightly sketch another median of the triangle. Now draw a median connecting the third vertex with the midpoint of the opposite side. Notice that the three lines meet at a common point, called

the "center of gravity" of the triangle.

Experiment 12—Make an isosceles triangle (two sides equal) by inserting the peg into the appropriate hole. Notice that the median is now perpendicular to the base, and that the two triangles are congruent, each being the mirror image of the other. Lightly sketch in the other medians. What about their lengths?

MIDPOINTS OF SIDES

Experiment 13—Preliminary to assembling this model, back the cardboard with corrugated board and paint black the wooden rod or dowel. At B and M punch holes just large enough to take the elastic. Thread the larger piece of elastic through B and through the hole at the lower end of the dowel. Knot the end of the elastic at the back of the model and at the under side of the dowel.

Thread the smaller elastic through M and through the hole at the midpoint of the dowel, pushing it into the dowel from the same side as the longer elastic, and knot both ends. Insert the carpet nail through the third hole at the upper end of the dowel, and punch it through the cardboard at A. With a pair of pliers bend the end of the nail so it does not

pull loose and secure with sealing wax. Rotate the dowel to be sure it pivots around the nail.

Experiment 14—Rotate the dowel to the right of the line AMB and notice the relationship of the lines formed by the two elastics, one making the base of the triangle and the other connecting the midpoints of the other two sides. Use the push pin from the first model to hold the dowel and measure the perpendicular distance between the two elastics.

This model illustrates the theorem: If a line joins the midpoints of two sides of a triangle, it is parallel to the third side. This is a special case of the theorem: If a line divides two sides of a triangle proportionally, it is parallel to the third side.

Experiment 15—Notice that the triangle formed by A, M and the midpoint of the dowel is similar in shape to the triangle formed by A, B and the end of the dowel. Thus one method of creating similar triangles (where the corresponding angles are equal, but the corresponding sides are not necessarily equal in length) is to divide two sides of a triangle proportionally and connect these two points. Another method obviously is to draw a line parallel to one of the sides.

CIRCUMSCRIBING A CIRCLE ABOUT A TRIANGLE

Experiment 16—Back the square of cardboard with corrugated board and color the circle yellow. Punch holes at each end of the chord (straight line connecting two points on the circumference of the circle), and thread the elastic through them. Knot the ends of elastic at the back of the model, leaving a loop about three-and-a-quarter inches long on the front side. Mark the exact center of the circle, inside the "O".

Experiment 17—Loop the elastic around the push pin and stick the pin into the cardboard at various points around the circle, noting when you form acute triangles, when obtuse triangles, when right triangles and when an isosceles triangle. Where does the center of the circle circumscribed about the various triangles lie in each case?

Experiment 18—Draw acute, obtuse, right and isosceles triangles on a piece of paper and try to guess where to locate the center of a circle that would pass through the three vertices. Now actually draw the circle you visualized. Did you guess correctly?

If you do not have a compass, make a

loop of thread as long as the radius of your proposed circle. Slip a pencil through one end of the loop and through the other stick the pin, pushing it into the spot chosen for locating the center of the circle. Draw a line, holding the string between pencil and pin taut.

Experiment 19—On a piece of paper draw a large triangle. At the midpoint of each side erect a line perpendicular to that side. Use the point where these three lines cross as the center of a circle. When the circle passes through one of the vertices of the triangle, does it pass through them all?

Experiment 20—Now form a right triangle by pushing the pin in the appropriate point on the circle. Note that one side of the triangle cuts directly across the center of the circle and thus is actually a diameter of the circle with the center of that circle at its midpoint. This line is also the hypotenuse of the triangle, for by definition the hypotenuse is the side opposite the right angle. Draw the median from the vertex of the right angle to the hypotenuse. It is a radius of the circle and thus only half as long as the hypotenuse.

Experiment 21—Hold the push pin so that both of the elastic sides of the triangle

lie on the same side of the center of the circle. Notice that one of the angles of the triangle is greater than 90 degrees. Form other such triangles to demonstrate that all triangles inscribed in an arc less than a semicircle are obtuse.

MAKE LARGE MODELS

In case you wish to demonstrate principles of geometry to a group of students or friends, large permanent models may be based upon these small ones. A construction board known as tempered hardboard makes the best base for your demonstration model, but wallboard, plywood, linoleum or extra heavy cardboard may be used. Elastic such as milliners use in ladies' hats has enough snap to serve as the triangle sides.

Experiment 22—For the "Triangles of Equal Area" model, use a rectangle of hardboard a foot wide and two feet long. Paint it orange or light red. If hardboard is not available, cut one of the other materials listed to the appropriate size and cover one side with orange cardboard. Seal the two together carefully with tape around the four edges.

Paint or draw two parallel lines seven-and-a-half inches apart across the entire

length of the board, using a color that will contrast nicely with the background paint and dark elastic. Paint a triangle with its ten-inch base as part of the lower one of the parallel lines and with its vertex lying on the upper line. At the end points of the base of the triangle bore small holes through which you can lace the elastic. A nail or screw will help make these holes if you do not have better tools. Thread a piece of elastic about 22 inches long through to the back side and knot securely.

If you used hardboard for this model, at intervals along the upper line drive small, short, round-headed screws. These screws should project 1/16 to 1/8 inch above the face of the model so the elastic can be looped over them. Screws will not hold firmly in cardboard, so use long pointed push pins should you use this material.

The model is now ready to demonstrate that triangles with equal bases and equal altitudes are always equal in area.

Experiment 23—A rectangular piece of hardboard 16 by 18 inches is about right for the "Median of a Triangle" model. Again paint one side orange or light red. Several inches from one long side paint or draw a black base line parallel to the side and about a foot long. Lo-

cate and mark with a pencil the vertices of two obtuse triangles, two acute triangles, a right triangle and an isosceles triangle.

At the end points and at the midpoint of the base line bore holes just large enough to take the elastic. Find a wooden rod about a quarter-inch thick and cut off a rod an inch-and-a-half long. Insert a small screw eye into the midpoint of the peg thus made. Tie one end of a piece of light-colored elastic to the eye of the screw and thread the other end through the hole at the midpoint of the base line. The points located for the vertices of the various triangles will determine how long the elastic should be.

Thread a long black elastic through the eye of the screw, then lace each end through a hole at the end point of the base line and tie at the back of the model. Test to be certain the points marking the vertices of the angles were well-placed, then at these points bore holes just large enough for the peg to fit snugly into them.

Experiment 24—The rectangle of hardboard used for the "Midpoints of Sides" model should be 14 by 24 inches. Paint it some pastel shade and draw a black line about 16½ inches long to the left side of the rectangle, slanting it some-

what like the line in your smaller model.

Bore small holes for the elastic at the ends and the midpoint of the line. From a quarter-inch dowel cut a piece 13½ inches long and paint black. Bore small holes about a quarter inch from each end of the wooden rod or dowel, and a hole midway between them. Insert a small bolt through an end hole of the dowel and through the hole in the board at the upper end of the line, then secure with a nut at the back of the model. Test to be sure the dowel turns easily.

Thread a black elastic through the midpoints of the line and of the dowel; thread another through the end points of the line and the dowel. Knot the elastic at the back of the model and at the under side of the dowel, making the end elastic the longer and leaving enough elastic to make a wide variety of figures. You are now ready to make all kinds of triangles by holding the dowel in various positions.

Experiment 25—A piece of hardboard 14 inches square will be fine for this demonstration model, "Circle around Triangle." Locate the center of the board

and mark carefully. Using this point as the center, draw a circle six inches in radius. Paint the circle yellow and the rest of the board some contrasting color.

Draw or paint in black a chord about eight and a half inches long and drill holes at the two end points. Thread elastic through these and knot on the back side, leaving a loop of elastic about 17 inches long to work with. In the hardboard drive small, short, round-headed screws along the circle to locate the vertices of several acute triangles, obtuse triangles, two right triangles and an isosceles triangle. The screws should project 1/16 to 1/8 inch above the face of the model so the elastic can be looped over them.

Your model is now ready for experiments such as those suggested for the smaller model.

Because of the enthusiastic response and the continued demand for the Geometric Models Unit, which was originally issued in April, 1949, this unit is being re-issued.

Issued March 1962

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12 monthly experimental kits—$5.00
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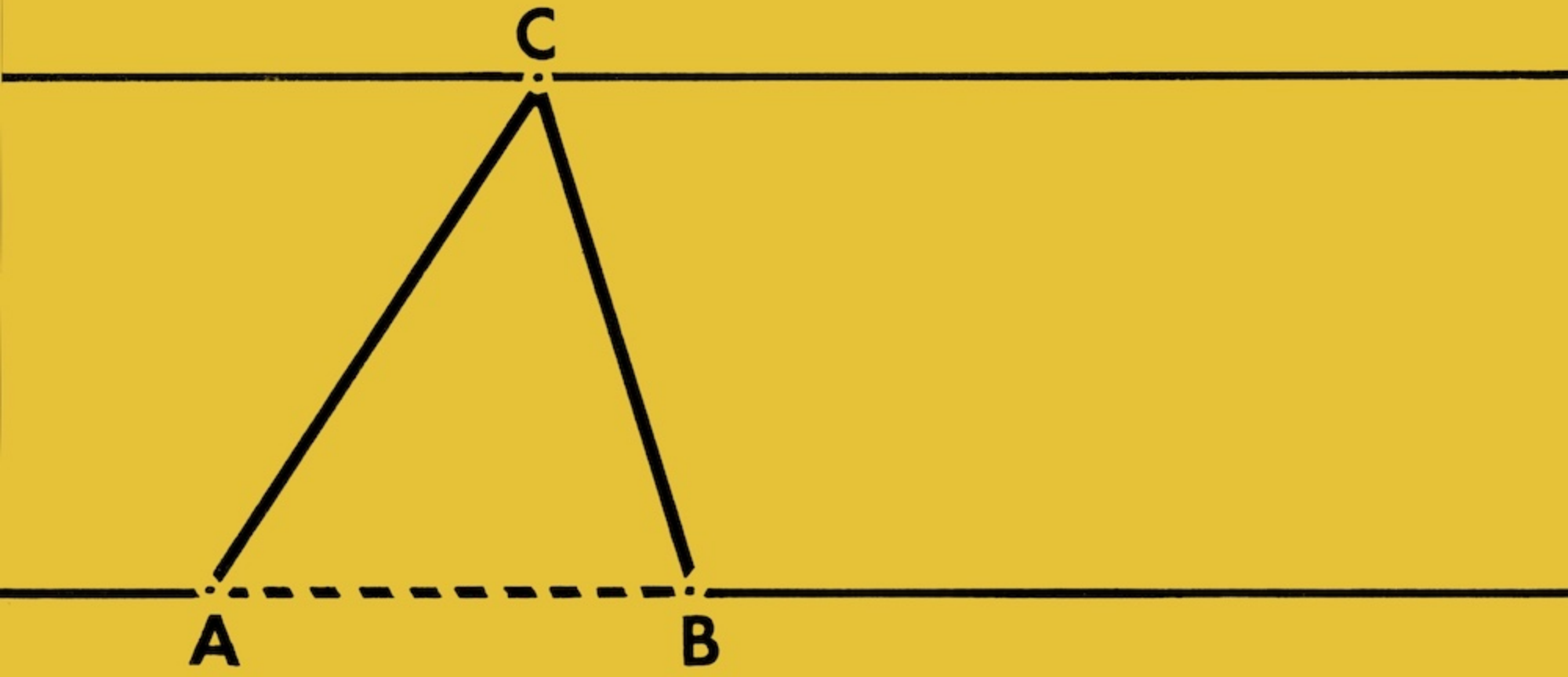
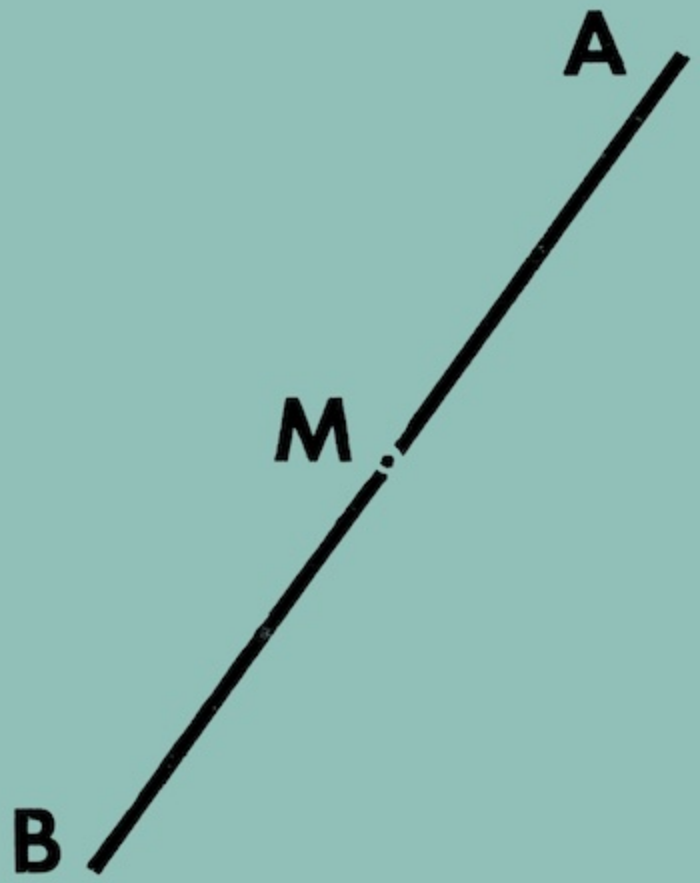


Triangles of Equal Area



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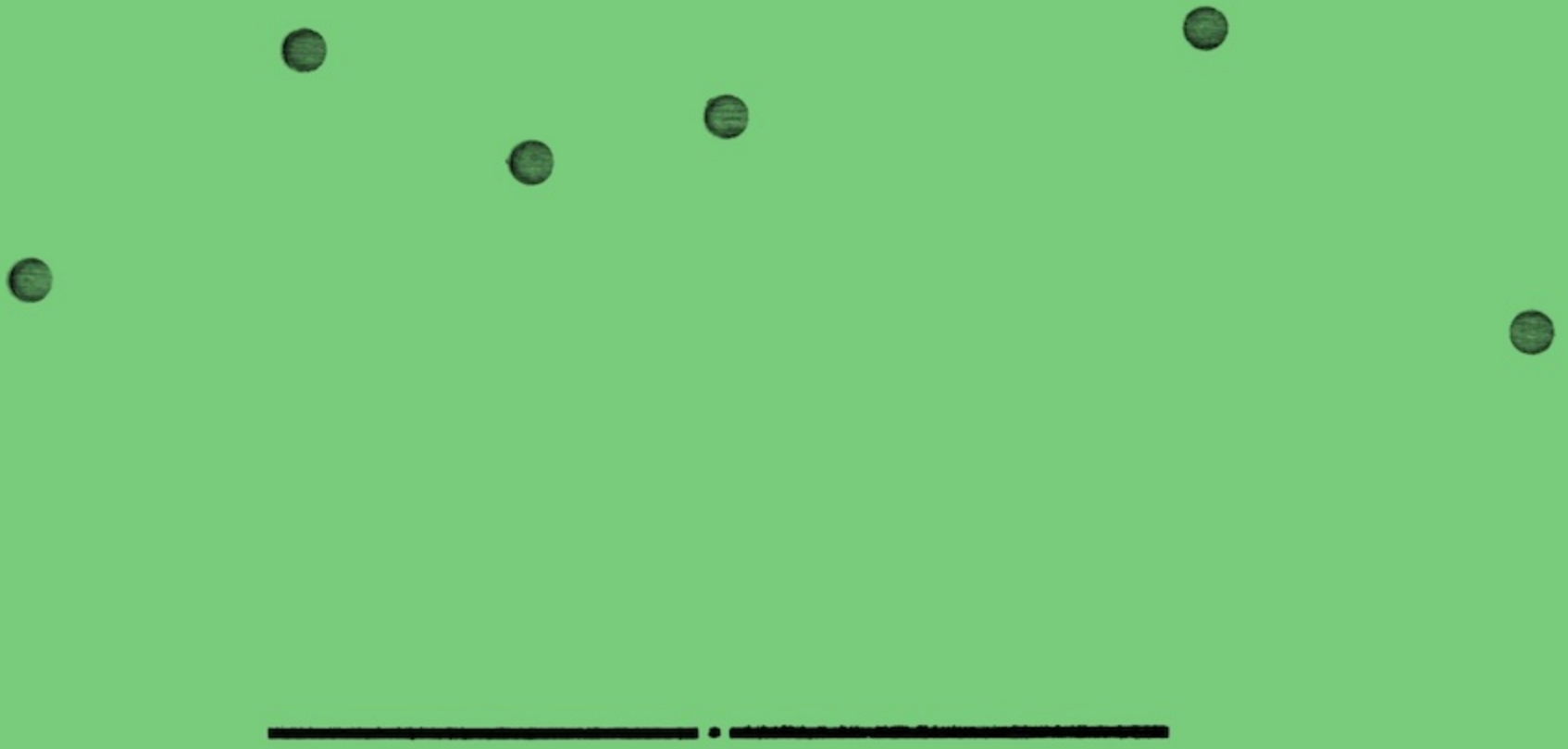
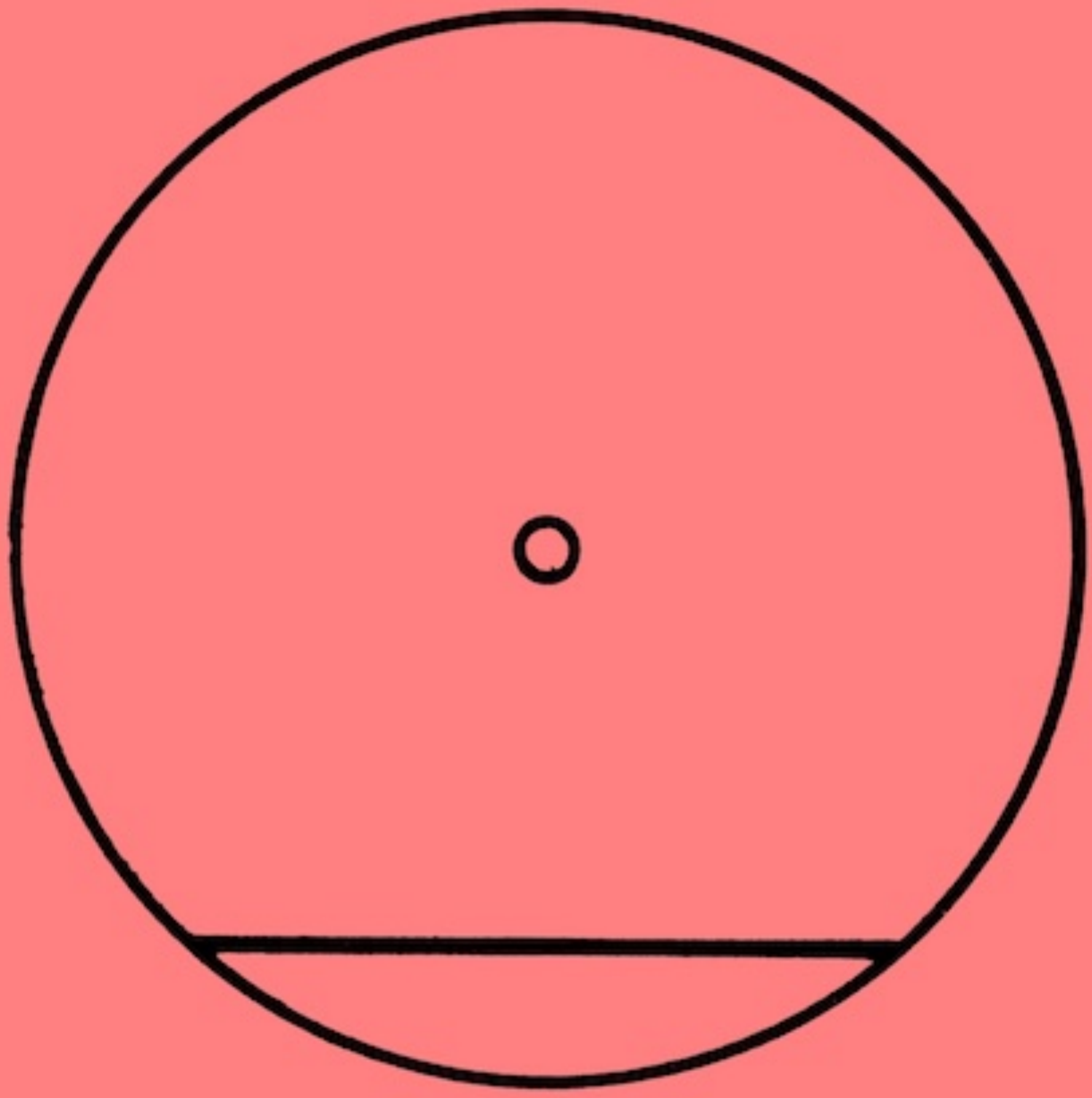


Median of a Triangle



Circle around Triangle